

vdash.org: a formal math commons

```
theorem ( a :: int )
dvd b ==> a ^ n dvd
b ^ n proof - assume
a dvd b show a ^ n
dvd b ^ n proof
( induct n ) show
a^0 dvd b^0 proof
- have a^0 = 1 by ( rule power-0 ) moreover
have ( 1 dvd b ^ 0 ) by ( rule zdvd-1-left )
ultimately show ?thesis by simp qed next
fix n assume a ^ n dvd b ^ n show a ^ Suc
n dvd b ^ Suc n proof - from prems
have a * a ^ n dvd b * b ^ n by ( intro
zdvd-zmult-mono ) moreover have a ^ Suc n
= a * a^n by ( rule
power-Suc ) moreover
have b ^ Suc n =
b * b^n by ( rule
power-Suc)ultimately
show ?thesis by simp
qed qed qed
```

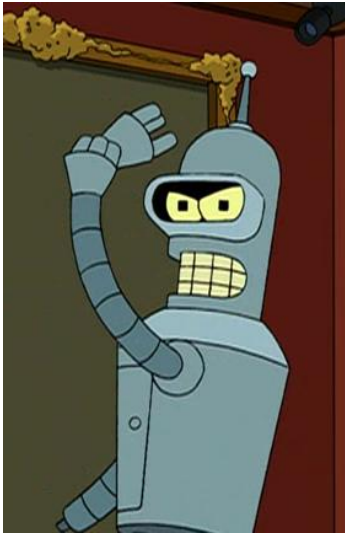
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“I demand satisfaction!”

What if Wikipedia allowed only true statements?

vdash

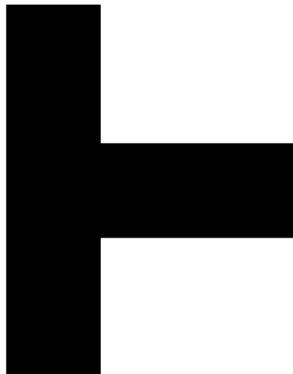
- wiki of formally verified math
- one road towards a “Math Commons”

vdash = interactive theorem prover
+ computer-checked math library
+ web interface

`\vdash` is \vdash in L^AT_EX.

$A \vdash B$

if B can be
proved from A .





Cantor's Theorem

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theorem $\exists S.S \notin \text{range } (f :: 'a \Rightarrow 'a \text{ set})"$

proof

```

let "?S = {x.x \notin fx}"
show "?S \notin range f"
proof
  assume "?S \in range f"
  then obtain y where fy: "?S = fy" ..
  show False
  proof cases
    assume "y \in ?S"
    hence "y \notin fy" by simp
    hence "y \notin ?S" by (simp add:fy)
    thus False by contradiction
  next
    assume "y \notin ?S"
    hence "y \in fy" by simp
    hence "y \in ?S" by (simp add:fy)
    thus False by contradiction
  qed
qed
qed

```



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proof

let " $?S = \{x.x \notin fx\}$ "

show " $?S \notin \text{range } f$ "

proof

assume " $?S \in \text{range } f$ "

then obtain y where fy : " $?S = fy$ " ..

show *False*

proof cases

assume " $y \in ?S$ "

hence " $y \notin fy$ " by *simp*

Why *Formalized* Math?

- Certainty
- Complete explanation
- Modularity & reusability
- Instant verification
- Interactivity

Knowledge Base for Robot Mathematicians?



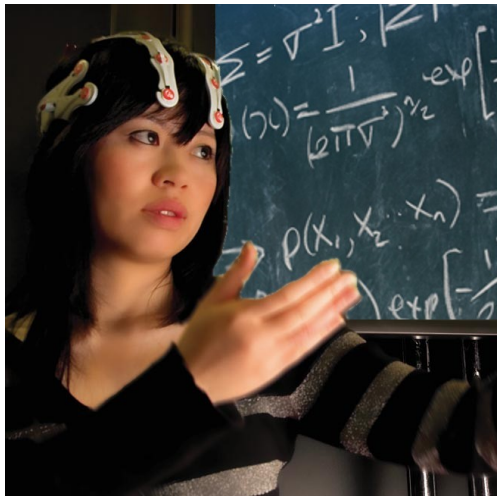
Data, Lucasian Professor of Mathematics

Knowledge Base for Robot Mathematicians?

“We are not scanning all those books to be read by people,” explained one of my hosts after my talk. “We are scanning them to be read by an AI.”

– George Dyson,
on his 2005 visit to Google (edge.org)

In the meantime...



Interface for
human/computer
symbiont
mathematicians

Currently, formalization can be overwhelming.



Too much is missing

- The tools are good
- Not enough math yet

Why a *wiki*?

- Tons more contributors
- Top-down development
(proof outlines)

Isabelle/Isar:

- free software
- small trusted core
- human readable mathematics



Scrape web for mathematics

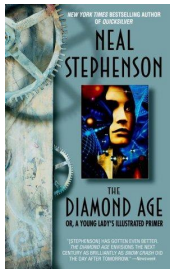
- arXiv, CiteSeer, other preprints
- open access journals

Connect to other sites

- Wikipedia, PlanetMath, MathWorld

Education

- interactive, open math textbooks
- “unfold” proofs all the way



LeActiveMath Grade 11

1 Introduction

2 What about the average

3 Difference quotients

4 Actual slope

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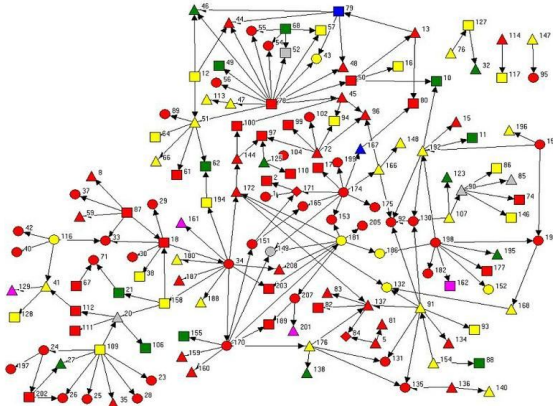
The slope in a point 8/40

$$m_{f, P_0} = \lim_{x \rightarrow x_0} m_{f, P_0} = \lim_{x \rightarrow x_0} \frac{y_x - y_0}{x_x - x_0} = \lim_{x \rightarrow x_0} \frac{f(x_x) - f(x_0)}{x_x - x_0}$$

The straight line that passes by P_0 with this slope m_{f, P_0} is called the **tangent** to the graph of f in the point P_0 .

The Theorème Project

visualize and browse interconnections
within mathematics



Industrial applications

- hardware, software, and protocol verification
- Zero-Knowledge Proofs?



How you can help

- Contribute math to the wiki
- Back-end
- User Interface
- Connections (scrape, link)
- Organize challenge projects

Learn more or
join the
mailing list at
vdash.org

Thanks!

