

# vdash.org: a formal math commons

```
theorem ( a :: int )
dvd b ==> a ^ n dvd
b ^ n proof - assume
a dvd b show a ^ n
dvd b ^ n proof
( induct n ) show
a^0 dvd b^0 proof
- have a^0 = 1 by ( rule power-0 ) moreover
have ( 1 dvd b ^ 0 ) by ( rule zdvd-1-left )
ultimately show ?thesis by simp qed next
fix n assume a ^ n dvd b ^ n show a ^ Suc
n dvd b ^ Suc n proof - from prems
have a * a ^ n dvd b * b ^ n by ( intro
zdvd-zmult-mono ) moreover have a ^ Suc n
= a * a^n by ( rule
power-Suc ) moreover
have b ^ Suc n =
b * b^n by ( rule
power-Suc)ultimately
show ?thesis by simp
qed qed qed
```

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“I demand satisfaction!”

What if Wikipedia allowed only true statements?

# vdash

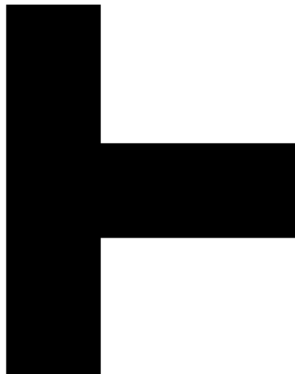
- wiki of formally verified math
- one road towards a “Math Commons”

vdash = interactive theorem prover  
+ computer-checked math library  
+ web interface

`\vdash` is  $\vdash$  in L<sup>A</sup>T<sub>E</sub>X.

$A \vdash B$

if  $B$  can be  
proved from  $A$ .





# Cantor's Theorem

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**theorem**  $\neg \exists S. S \notin \text{range } (f :: 'a \Rightarrow 'a \text{ set})"$

**proof**

```

let "?S = {x.x \notin fx}"
show "?S \notin range f"
proof
  assume "?S \in range f"
  then obtain y where fy: "?S = fy" ..
  show False
  proof cases
    assume "y \in ?S"
    hence "y \notin fy" by simp
    hence "y \notin ?S" by (simp add:fy)
    thus False by contradiction
  next
    assume "y \notin ?S"
    hence "y \in fy" by simp
    hence "y \in ?S" by (simp add:fy)
    thus False by contradiction
  qed
qed
qed

```



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**proof**

let "  $?S = \{x.x \notin fx\}$  "

show " $?S \notin \text{range } f$ "

**proof**

assume " $?S \in \text{range } f$ "

then obtain  $y$  where  $fy$ : " $?S = fy$ " ..

show *False*

**proof** cases

assume " $y \in ?S$  "

hence " $y \notin fy$  " by *simp*

# Why *Formalized* Math?

- Certainty
- Complete explanation
- Modularity & reusability
- Instant verification
- Interactivity

# Knowledge Base for Robot Mathematicians?



Data, Lucasian Professor of Mathematics



## Knowledge Base for Robot Mathematicians?

*“We are not scanning all those books to be read by people,” explained one of my hosts after my talk. “We are scanning them to be read by an AI.”*

– George Dyson,  
on his 2005 visit to Google ([edge.org](http://edge.org))

In the meantime...



Interface for  
human/computer  
symbiont  
mathematicians

Currently, formalization can be overwhelming.



## Too much is missing

- The tools are good
- Not enough math yet

## Why a *wiki*?

- Tons more contributors
- Top-down development  
(proof outlines)

## Isabelle/Isar:

- free software
- small trusted core
- human readable mathematics



## Scrape web for mathematics

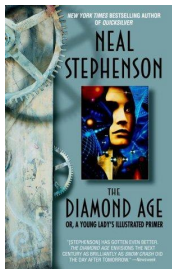
- arXiv, CiteSeer, other preprints
- open access journals

## Connect to other sites

- Wikipedia, PlanetMath, MathWorld

# Education

- interactive, open math textbooks
- “unfold” proofs all the way



LeActiveMath Grade 11

- 1 Introduction
- 2 What about the average
- 3 Difference quotients
  - The difference quotient (dynamic examples and exercises)
  - A difference quotient in a fixed rope route
- 4 Actual slope

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### The slope in a point

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$$m_{f, P_0} = \lim_{x \rightarrow x_0} m_{f, P_0, x} = \lim_{x \rightarrow x_0} \frac{y_x - y_0}{x_x - x_0} = \lim_{x \rightarrow x_0} \frac{f(x_x) - f(x_0)}{x_x - x_0}$$

The straight line that passes by  $P_0$  with this slope  $m_{f, P_0}$  is called the **tangent** to the graph of  $f$  in the point  $P_0$ .





# Industrial applications

- hardware, software, and protocol verification
- Zero-Knowledge Proofs?



# How you can help

- Contribute math to the wiki
- Back-end
- User Interface
- Connections (scrape, link)
- Organize challenge projects

Learn more or  
join the  
mailing list at  
[vdash.org](http://vdash.org)

Thanks!

