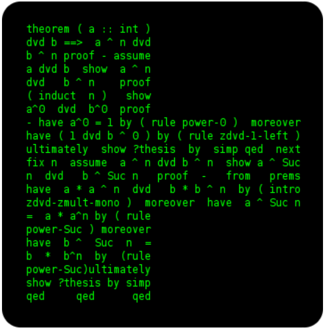
vdash.org:
a formalized math wiki



```
theorem ( a :: int )
dvd b ==>  a ^ n dvd
b ^ n proof - assume
a dvd b  show  a ^ n
dvd   b ^ n    proof
( induct  n )   show
a^0  dvd  b^0  proof
- have a^0 = 1 by ( rule power-0 )  moreover
have ( 1 dvd b ^ 0 ) by ( rule zdvd-1-left )
ultimately  show ?thesis  by  simp qed  next
fix n  assume  a ^ n dvd b ^ n  show a ^ Suc
n  dvd   b ^ Suc n   proof  -   from   prems
have  a * a ^ n  dvd   b * b ^ n  by ( intro
zdvd-zmult-mono )  moreover  have  a ^ Suc n
=  a * a^n by ( rule
power-Suc ) moreover
have  b ^  Suc  n  =
b  *  b^n  by  (rule
power-Suc)ultimately
show ?thesis by simp
qed     qed      qed
```

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MIT E-Club
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1. What is vdash?

2. Why?

3. Who?
    - Perspectives and communities

4. How?
    - Implementation
    - Screenshot
    - Future Developments
    - Contribute

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- one approach to developing the “Math Commons”

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The idea of vdash is to harness the collaborative power of a wiki, while retaining the absolute certainty of a mechanical verifier.

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"Sorry, this site allows only *true* statements."

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(In practice, probably allow any contribution, but indicate level of verification visually – e.g., with different colors of links.)

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- Knowledge base for robot mathematicians

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- Tool/interface for human + computer symbiont mathematicians

Biggest stumbling block: intermediate results

Why does LaTeX work?

Most mathematicians now use LaTeX to typeset their mathematics because it is easier than sending texts to secretaries and rechecking. Currently, using proof assistants is much harder than writing informally.

So try to develop libraries, tools, and interfaces to a point where it is easier to formally verify one's own results.

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Two main cases where formalization occurs today

- carefully curated bottom-up libraries
- vertical projects – do what it takes to get the big result

There are plenty of frameworks for the stuff in between, but it still is just tons of work.

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Hopefully, it is possible to build a community that produces way more formalized mathematics than with existing methods (which are careful, but too slow).

(Over time, ever larger "cores" could be cleaned up and submitted to existing careful forums.)

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Open content movements

- Science Commons
    - shared building-blocks for other collaborative sciences
- Creative Commons
- Free Culture

The *Formal* Math Encyclopedia that anyone can edit

- Wikipedia
- PlanetMath
- MathWorld

Better search through formalization

- Mathematics Knowledge Management
- Semantic Web

Existing formalization efforts

- Isabelle: Archive of Formal Proofs, IsarMathLib
- Mizar: Journal of Formalized Mathematics
- Coq: Coq Contribs, Hypertextual Electronic Library of Mathematics
- HOL: HOL Light examples

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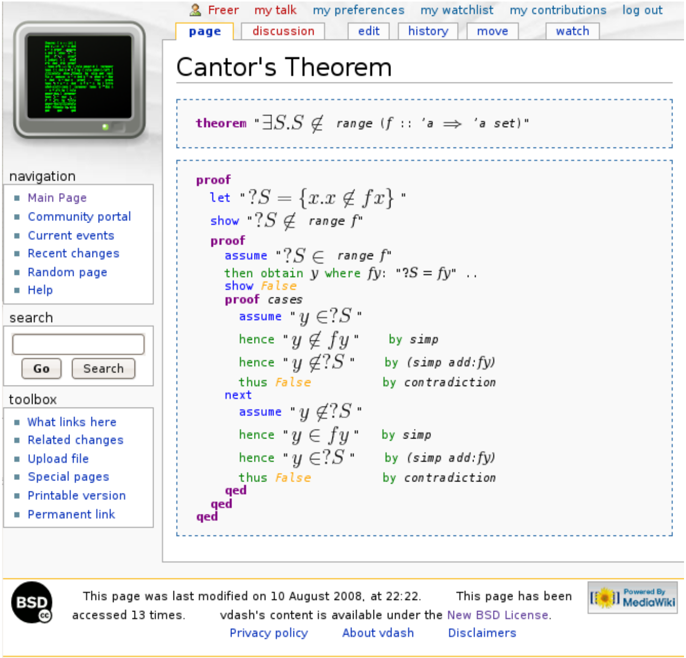
contributions under new BSD license



Cantor's Theorem

```
theorem "∃S. S ∉ range (f :: 'a ⇒ 'a set)"
```

```
proof
  let "?S = {x. x ∉ f x} "
  show "?S ∉ range f"
  proof
    assume "?S ∈ range f"
    then obtain y where fy: "?S = f y" ..
    show False
    proof cases
      assume "y ∈ ?S"
      hence "y ∉ f y"    by simp
      hence "y ∉ ?S"     by (simp add: fy)
      thus False         by contradiction
    next
      assume "y ∉ ?S"
      hence "y ∈ f y"    by simp
      hence "y ∈ ?S"     by (simp add: fy)
      thus False         by contradiction
    qed
  qed
qed
```

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Future features (not that the main site is ready yet!)

Scrape web for mathematics

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- open-source math textbooks

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clientside AJAX interface

- interactive verification
- better GUI
- reduce server load

Extensions and spinoffs

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Certified calculations and visualizations

- Computer algebra systems
- Graphing calculators and other technical graphics

New Domains

Formal science and engineering

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Industrial applications

- software and hardware verification
- more modular code

Doing the same thing with new content, or reusing building-blocks?

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Contribute math to the wiki

- formalize the statement of a theorem
- fill in proof details informally
- formalize details
- correct errors; make it prettier/idiomatic

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Back-end design

- versioning and dependencies of math code itself
- integrate bugtracker with Talk pages
- semantic metadata

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Interface to other projects

- web scrapers and internal bots
  - create vdash stubs
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Challenges

- Organize challenge projects and competitions to build up content and get people excited.

Feedback

Advice/ideas/suggestions: `freer@mit.edu`

Thanks!